arXiv:hep-th/0505150v1 17 May 2005

Fine structure constant in the spacetime of a cosmic string

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(February 1, 2008)

Abstract

We calculate the fine structure constant in the spacetime of a cosmic string. In the presence of a cosmic string the value of the fine structure constant reduces. We also discuss on numerical results.

The gravitational properties of cosmic strings are strikingly different from those of non-relativistic linear distributions of matter. To explain the origin of the difference, we note that for a static matter distribution with energy-momentum tensor,

$$T^{\mu}_{\nu} = \mathrm{diag}\left(\rho, -p_1, -p_2, -p_3\right), \tag{1}$$

the Newtonian limit of the Einstein equations become

$$\nabla^2 \Phi = 4\pi G \left(\rho + p_1 + p_2 + p_3\right), \tag{2}$$

where Φ is the gravitational potential. For non-relativistic matter, $p_i \ll \rho$ and $\nabla^2\Phi = 4\pi G\rho$. Strings, on the other hand, have a large longitudinal tension. For a straight string parallel to the z-axis, $p_3 = -\rho$, with p_1 and p_2 vanish when averaged over the string cross-section. Hence, the right-hand side of Eq.(2) vanishes, suggesting that straight strings produce no gravitational forece on surrounding matter. This conclusion is confirmed by a full general-relativistic analysis. Another feature distinguishing cosmic strings from more familiar sources is their relativistic motion. As a result, oscillating loops of string can be strong emitters of gravitational radiation.

A gravitating string is described by the combined system of Einstein, Higgs and guage field equations. The problem of solving these coupled equations is formidable and no exact solutions have been found to date. Fortunately, for most cosmological applications the problem can be made tractable by adopting two major simplifications. First, assuming that the string thickness is much smaller than all other relevant dimensions, the string can be

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approximated as a line of zero width with a distributional δ-function energy momentum tensor. Secondly, the gravitational field of the string is assumed to be sufficiently weak that the linearized Einstein equations can be employed [1]. These approximations are not valid for supermassive strings with a symmetry breaking scale $\eta \geq m_{\mathrm{P}}$. However, for strings with $\eta \ll m_{\mathrm{P}}$, linearized gravity is applicable almost everywhere, except in small regions of space affected by cusps and kinks. The analysis in this letter is based on these thin-string and weak-gravity approximations. The metric of a static straight string lying along the z-axis in cylindrical coordinates

$$ds^2 = dt^2 - dz^2 - (1-h)(dr^2 + r^2 d\theta^2), \tag{3}$$

where G is Newton's gravitational constant, μ the string mass per unit length and

$$h = 8G\mu \ln\left(\frac{r}{r_0}\right). \tag{4}$$

Introducing a new radial coordinate r' as

$$(1-h)r^2 = (1-8G\mu)r'^2, \tag{5}$$

we obtain to linear order in $G\mu$,

$$ds^2 = dt^2 - dz^2 - dr'^2 - (1-8G\mu)r'^2 d\theta^2, \tag{6}$$

Finally, with a new angular coordinate

$$\theta' = (1-4G\mu)\theta, \tag{7}$$

the metric takes a Minkowskian form

$$ds^2 = dt^2 - dz^2 - dr'^2 - r'^2 d\theta'^2. \tag{8}$$

So, the geometry around a straight cosmic string is locally identical to that of flat spacetime. This geometry, however is not globally Euclidean since the angle θ' varies in the range

$$0 \leq \theta' < 2\pi(1-4G\mu). \tag{9}$$

Hence, the effect of the string is to introduce an azimuthal 'deficit angle'

$$\Delta = 8\pi G\mu, \tag{10}$$

implying that a surface of constant t and z has the geometry of a cone rather than that of a plane [2].

As shown above, the metric (6) can be transformed to a flat metric (8) so there is no gravitational potential in the space outside the string. But there is a delta-function curvature at the core of the cosmic string which has a global effect-the deficit angle (10).

The dimensionless parameter $G\mu$ plays an important role in the physics of cosmic strings. In the weak-field approximation $G\mu \ll 1$. The string scenario for galaxy formation requires $G\mu \sim 10^{-6}$ while observations constrain $G\mu$ to be less than 10^{-5} [2].

The authors of Refs. [3, 4, 5] have shown that the electrostatic field of a charged particle is distorted by the cosmic string. For a test charged particle in the presence of a cosmic string the electrostatic self-force is repulsive with magnitude

$$F = \frac{\pi G\mu e^2}{4r^2}. \tag{11}$$

For the Bohr's atom in the absence of a cosmic string, the electrostatic force between an electron and a proton is given by Coulomb law:

$$F = \frac{-ke^2}{r^2}, \tag{12}$$

where $k = 1/(4\pi\epsilon_0) = 8.99 \times 10^9 \mathrm{N.m^2/C^2}$. The orbital speed of the electron in the first Bohr orbit is

$$v_1 = \frac{ke^2}{\hbar}. \tag{13}$$

The ratio of this speed to the speed of light, v_1/c, is known as the fine structute constant [6]:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}. \tag{14}$$

Inserting the known values of the constants in the right-hand side of this equation shows that $\alpha = 1/137.0360$. Thus the electron in the first Bohr orbit moves at $\frac{1}{137}$ the speed of light.

To calculate the fine structure constant in the spacetime of a cosmic string we consider a Bohr's atom in the presence of a cosmic string. For a Bohr's atom in the spacetime of a cosmic string, we take into account in Eq.(11) the sum of two forces, i.e. the electrostatice force for Bohr's atom in the absence

of a cosmic string, given by Eq.(12), plus the electrostatic self-force of the electron in the presence of a cosmic string. Therefore, we have

$$F = -\frac{e^2}{4\pi\epsilon_0 r^2} + \frac{\pi G\mu e^2}{4r^2}. \tag{15}$$

It can be easily shown that this force has negative value and is an attractive force ($\pi^2\epsilon_0 G\mu < 1$).

The numerical value of the fine structure constant in the spacetime of a cosmic string can be computed by Eq.(15). The orbital speed of the electron in the first Bohr orbit in the spacetime of a cosmic string has positive value and is given by:

$$\hat{v}_1 = \frac{e^2}{4\pi\epsilon_0\hbar} - \frac{\pi G\mu e^2}{4\hbar}. \tag{16}$$

The ratio of this speed to the speed of light, $\hat{v}_1/c$, is presented by the symbol $\hat{\alpha}$ which is the fine structure constant in the spacetime of a cosmic string:

$$\hat{\alpha} = \frac{e^2}{4\pi\epsilon_0\hbar c} - \frac{\pi G\mu e^2}{4\hbar c}. \tag{17}$$

From (14) and (17) we obtain:

$$\frac{\alpha}{\hat{\alpha}} = \frac{1}{1-\pi^2\epsilon_0 G\mu}. \tag{18}$$

In the limit $G\mu \to 0$, i.e. in the absence of the cosmic string, $\alpha/\hat{\alpha} \to 1$. From Eq.(18) we obtain $(\alpha - \hat{\alpha})/\alpha = \pi^2\epsilon_0 G\mu$. Inserting $G\mu \simeq 10^{-6}$ and the known values of the constants in the right-hand side of Eq.(17) yields

$$\hat{\alpha} = \left(1 - 8.736 \times 10^{-17}\right)\alpha. \tag{19}$$

This means that the presence of a cosmic string causes the value of the fine structure constant reduces. This reduction in the value of the fine structure constant is very small, as given in Eq.(19).

Acknowledgments: F.N. thanks Amir and Shahrokh for useful helps.

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